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TITLE: NEUTRINO OSCILLATIONS IN THE EARLY UNIVERSE II

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Neutrino Oscillations in the Early Universe II.

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ABSTRACT

We show that it is possible for neutrino oscillations to create a state of the universe in which the net lepton numbers are appreciable and different for each species at the time of nucleosynthesis. Under these conditions mixing induced by the oscillations can influence the abundances of the light elements.

INTRODUCTION

At the first Telemark Neutrino Conference we presented preliminary results on the influence of neutrino oscillations and mixing on the cosmological production of ^{4}He.[1] We promised further results on the cosmological production of the light elements and also a study of the effects of neutrino oscillations during the non-equilibrium period when the oscillation length is of the order of the size of the universe. Now we make good on these promises.

To understand the possible effects neutrino oscillations may have during the early stages of cosmology, it is important to understand the four characteristic times associated with the neutrinos. These are the inverse of the rate of expansion of the universe

$$t \sim \frac{1}{\sqrt{G_N} T^2} \quad , \tag{1}$$

the mean time between left-handed neutrino interactions,

$$\tau_L \sim \frac{1}{G_F^2 T^5} \sim \frac{(\sqrt{G_N} t)^{5/2}}{G_F^2} \quad , \tag{2}$$

the corresponding time for right-handed neutrino interaction τ_R, which, since $m_{W_R} >> m_{W_L}$, is much greater that τ_L, and the oscillation time[2]

$$t_0 \sim \frac{1}{\Delta m^2} \cdot \frac{1}{\Delta m^2 (\sqrt{G_N} t)^{\frac{1}{2}}} \quad . \tag{3}$$

As the universe cools down, τ, τ_2 and τ_R all increase, whereas t_0 decreases, as is shown in figure 1. At small enough t, $t_0 >> t$ and the oscillations have no time to develop and thus exert no cosmological influence. There is always a time t_0^* at which $t_0 = t$, but whether the oscillations have any influence depends on whether t_0^* is greater or less that τ_L^*, the left-handed neutrino decoupling time at which $\tau_L = t$. If $t_0^* > \tau_L^*$, then by the time that the neutrino oscillations are able to develop fully the neutrinos have already decoupled and the oscillations cannot influence the evolution of the universe. However if $t_0^* < \tau_L^*$ as illustrated in figure 1, then there is a stage at which the oscillation time is of the order of the expansion times while being greater than τ_L. At this stage the oscillations are a non-equilibrium perturbation on the time evolution of the universe. If we introduce a general neutrino mass matrix, and include CP non-conservation in the lepton section we have the classic requirements for generating a lepton excess - a point which was made by Kholopov and Petrov.[3]

We have investigated the non-equilibrium problem in more detail, and find that the effect discussed by Kholopov and Petrov[3] in fact allows a different lepton excess for each species of lepton. This implies that it is possible for the different species of neutrinos to have different but appreciable chemical potentials. It was this circumstance we showed had significant effects on the ^{4}He generation in our earlier paper, provided that $t_0 << \tau$ near $t \sim \tau^*$, as is the case in figure 1.

The condition that $t_0^* < \tau_L^*$ for oscillations to be potentially interesting requires

$$\Delta m^2 > G_N/G_F^2 \sim 10^{-9} eV^2 \tag{4}$$

showing that cosmology is able to probe very small mass differences between neutrinos.

In this paper we will first set up the formalism, and then describe in detail the non-equilibrium processes which generate these interesting conditions. Finally we present our results on nucleosynthesis of light elements when these conditions prevail near the decoupling time.

II. FORMALISM

We introduce a set of weak eigenstates $\nu_{\alpha L}$ which participate in the usual left-handed weak interactions, appearing in the charged current as $\bar{\ell}_\alpha \gamma_\mu (1+\gamma_5) \nu_{\alpha L}$ where ℓ_α is the charged lepton and $\alpha = e, \mu, \tau, \ldots, \ell_{n_f}$. In addition we allow for right-handed neutrinos

$\nu'_{\alpha R}$ which participate in the much weaker right-handed weak interactions. The mass matrix is then of the general Majorana + Dirac form

$$= \sum_{\alpha,\beta} \Big[M_{\alpha\beta} \overline{(\nu_{\alpha L})} \nu^c_{\beta L} + m_{\alpha\beta} \overline{(\nu'_{\alpha R})^c} \nu'_{\beta R}$$

$$+ D_{\alpha\beta} \left\{ \overline{(\nu_{\alpha L})} \nu'_{\beta R} + \overline{(\nu'_{\beta R})^c} \nu^c_{\alpha L} \right\} + \text{h.c.} \Big] \quad . \tag{5}$$

The diagonalization of mixes $\nu_{\alpha L}$ and $\nu_{\beta L}$, and also $\nu_{\alpha L}$ and $(\nu'_{\beta R})^c$. In the usual terminology not only are the left-handed neutrinos mixed, but the left-handed neutrinos are also mixed with left-handed antineutrinos.

The mass eigenstates are Majorana particles. If we designate these eigenstates by N_i where $i = 1,2,\ldots,2n_f$, and set

$$\nu_{(\alpha+n_f)L} = (\nu'_{\alpha R})^c \quad , \tag{6}$$

then we can regard the ν_γ as obtained from the mass eigenstates by a unitary transformation

$$\nu_{\gamma L} = U_{\gamma i} N_i \quad . \tag{7}$$

Oscillations, of both the first and second classes, may then be obtained in the usual way.[2]

It is, however, more convenient to introduce the density matrix $\underline{f}(t)$, which is such that $\langle\alpha|\underline{f}(t)|\alpha\rangle$ is the number of neutrinos in the state α at time t, given the inital number of neutrinos $\langle\alpha|\underline{f}(0)|\alpha\rangle$ of each species. To allow for expansion of the universe, and for collisions we replace the usual evolution equation for $\underline{f}$

$$\frac{d\underline{f}}{dt} = i[\underline{f},\underline{H}_0] \quad , \tag{9}$$

(where $\underline{H}_0$ is the Hamiltonian for free neutrinos) with the Boltzman type of equation

$$\frac{d\underline{f}}{dt} - \frac{3\dot{R}}{R}\underline{f} = i[\underline{f},\underline{H}] + \underline{C}[\underline{f}] \quad , \tag{10}$$

where the matrix $\underline{C}$ describes the collision terms, which tend to maintain $\underline{f}$ in the equilibrium distribution. Our attempts to const ict detailed inter ing solutions to (10) are still in progress (perhaps to be reporte Telemark III?), but we believe that some qualitative statements be made, following Kholopov and Petrov.[3] This we now do.

III. THE NON-EQUILIBRIUM PHASE

We first note that the right-handed neutrinos interact much more weakly than the left-handed neutrinos, and so decouple at an earlier time τ_R^*. As a result they do not participate in any subsequent heating of the universe that occurs because of annihilation of particles and antiparticles of mass m as T drops below m. Thus T_R, the temperature of the right-handed neutrinos will be related to T_L by

$$\frac{T_R}{T_L} = \left(\frac{4}{11}\right)^{s/3}$$

where s is the number of species which have annihilated between τ_R^* and t.

If $\tau_R^* << \tau_L^*$ then $T_R << T_L$ and the number density of right-handed neutrinos (or left-handed antineutrinos) will be much less than that of the left-handed neutrinos. This is the driving force which, coupled with oscillations, produces a net neutrino number for the universe.

In general, the neutrino densities are given by

$$n[\nu_{\alpha L}] = \sum_{\gamma=1}^{2n_f} \langle n[\nu_{\gamma L}]P(\nu_{\gamma L}\rightarrow\nu_{\alpha L})\rangle_E \tag{11}$$

where the average is over the energy distribution of the neutrinos and $P(\nu_{\gamma L} \rightarrow \nu_{\alpha L})$ is the transition probability which strictly should be obtained from the Boltzman equation (10), or an equivalent master equation. Since we are assuming $T_R << T_L$,

$$n[\nu_{\alpha+n_f,L}] << n[\nu_{\alpha,L}] \tag{12}$$

and we may truncate the sum in (12) at n_f, obtaining

$$n[\nu_{\alpha,L}] = \sum_{\gamma=1}^{n_f} \langle n[\nu_{\gamma,L}]\ P(\nu_{\gamma L}\rightarrow \nu_{\alpha L})\rangle_E \quad . \tag{13}$$

However, unitarity requires

$$\sum_{\gamma=1}^{2n_f} P(\nu_{\gamma L}\to\nu_{\alpha L}) = 1 \tag{14}$$

which allows

$$\sum_{\gamma=1}^{n_f} P(\nu_{\gamma L}\to\nu_{\alpha L}) \neq 1 \tag{15}$$

in the presence of second class oscillations. Thus even if we start with $n[\nu_{\gamma L}] = n[\nu_{\alpha L}]$ for $\alpha,\gamma, \leq n_f$, equation (13) does not preserve this equality. We thus obtain our first lemma:

> Lemma 1: The number densities of the different neutrino species will in general be different in the presence of neutrino oscillations of the second class.

The antiparticle densities $n[(\nu_{\alpha L})^c]$ satisfy an equation analogous to (13),

$$n[(\nu_{\alpha L})^c] = \sum_{\gamma=1}^{n_f} \langle n[(\nu_{\gamma L})^c]\ P\ ((\nu_{\gamma L})^c\to(\nu_{\alpha L})^c)\rangle_E \tag{16}$$

In general the initial conditions will be such that $n[(\nu_{\gamma L})^c] = n[(\nu_{\gamma L})]$; there is no net lepton number of the universe (see however, Harvey and Kolb[4] for a scenario where this is not true in general).

If however, we have CP violation as well as second class oscillations then

$$P(\nu_{\gamma L}\to\nu_{\alpha L}) \neq P((\nu_{\gamma L})^c\to(\nu_{\alpha L})^c) \quad . \tag{17}$$

Let us introduce the net lepton density of the α species, L_α, defined by

$$L_\alpha = n[\nu_{\alpha L}] - n[(\nu_{\alpha L})^c] \tag{18}$$

which then satisfies the equation

$$L_\alpha = <\sum_{\gamma=1}^{n_f} L_\gamma P(\nu_{\gamma L}\to\nu_{\alpha L})$$

$$+ n[(\nu_{\gamma L})^c]\cdot[P(\nu_{\gamma L}\to\nu_{\alpha L}) - P((\nu_{\gamma L})^c\to(\nu_{\alpha L})^c)]>_E \quad . \tag{19}$$

So even if, initially, $L_\gamma = 0$ and $n[(\nu_{\gamma L})^c] = n[(\nu_{\alpha L})^c]$ for γ and $\alpha \leq n_f$, the second class oscillations can develop $L_\gamma \neq 0$, and $L_\gamma - L_\alpha \neq 0$.

Thus we arrive at our second and third lemmas:

<u>Lemma 2: In the presence of second class oscillations and CP violations, generations of appreciable net lepton numbers L_α is possible.</u>

<u>Lemma 3: In the presence of second class oscillations and CP violation, the lepton numbers of different neutrino species need not be equal.</u>

Lemma 2 is the result of Khopolov and Petrov, but lemmas 1 and 3 are, to our knowledge, new.

We proceed to remark that if these oscillations occur while the left-handed neutrinos are still in thermal equilibrium with the universe, then these number distributions will be thermalized, and will thus be characterized by the equilibrium temperature T, and a chemical potential μ_α which will in general be nonzero and different for different species.

We note that the experimental limit on μ is

$$\xi_\alpha = \frac{\mu_\alpha}{T} \leq 60$$

as long as the neutrinos are light (of rest mass less than 1 meV). This limit applies to all species and is simply a consequence of requiring that the density of background neutrinos not exceed the critical density. Values of ξ_α or order 1 are not excluded by observational evidence.

After this phase $t \sim t_0^* > \tau_L$, the oscillation period decreases so that $t_0 \ll t, \tau_L$. At this stage the oscillations simply develop an equilibrium distribution characterized by

$$[\underline{\xi}, \underline{U}] = 0 \ , \tag{20}$$

to eliminate term of order t_0^{-1} which would otherwise dominate the Boltzman equation, and

$$\underline{\underline{C}}[\underline{\underline{f}}] = 0 \tag{21}$$

which eliminates the term in τ_L^{-1}.

(20) requires that $\underline{\underline{f}}$ the diagonal in the mass basis, and (21) that $\underline{\underline{f}}$ be an equilibrium distribution. We examine the form and consequences of this equilibrium distribution in the next section.

IV. THE EQUILIBRIUM SITUATION

We have already described the circumstances in which $\underline{\underline{f}}$ takes on the equilibrium form for the left-handed neutrinos. In general $\underline{\underline{f}}$ is diagonal in the mass basis, and

$$f_{ij} = f_i \delta_{ij} \tag{22}$$

with $f_i \propto \dfrac{1}{e^{(E_i - \mu_i)/kT} + 1}$ (23)

with $\mu_i \neq \mu_j$.

It is then easy to compute the reaction rate, e.g., for $\nu_e n \to ep$, for which the T matrix element is T_{ee}, where the subscripts refer to lepton states in the weak interaction basis. Then

$$\Gamma(\nu_e n \to ep) = (T\underline{\underline{f}}T)_{ee} \tag{24}$$

$$= \sum_i |T_{ee}|^2 |U_{ei}|^2 f_i \quad . \tag{25}$$

If either the μ_i are different for different i, or there are second class oscillations, or both, then rates computed through (25) will differ from those without neutrino mixing.

We have set up the network equations up to ^{4}He and solved them for the light element abundances for a simple two neutrino species model, say ν_e and ν_μ, ignoring for this stage of the calculation the right-handed neutrinos. In this case there are three additional parameters in the problem and it should be no surprise that we can obtain good fits to the ^{4}He, ^{3}He and d abundances, even if the limit $N_\nu \lesssim 4$ is relaxed. For example, with $\Theta = 30°$, we obtain $Y_{He} \sim 0.21$ for $N_\nu = 6$ at $\eta = 3 \times 10^{-10}$ for $\xi_1 = 0.3$, $\xi_2 = 0.5$. These same parameters then give

$$\frac{X_d}{X_H} \sim 3 \times 10^{-5} \text{ and } \frac{X_{^3He}}{X_H} \sim 4 \times 10^{-5} \quad ,$$

which are in good agreement with observations. Detailed results will be published elsewhere, but it should be obvious that there is now more than enough freedom in the parameters to fit the observed abundances without serious restriction on N_ν. Indeed, for nucleon to photon ratios smaller that 10^{-9} the sensitivity of X_d and $X_{^3He}$ to ξ_1 and ξ_2 is weak enough that any pair of values of ξ_1 and ξ_2 which fit $Y_{^4He}$ will also fit X_d and $X_{^3He}$.

V. CONCLUSIONS

We have shown that if second class oscillations and CP violations are introduced in the neutrino sector it is possible to generate a situation where the lepton species numbers L_α are appreciable and distinct at the time of nucleosynthesis. These are precisely the conditions under which neutrino mixing induced by neutrino oscillations can influence the nucleosynthesis process. These circumstances introduce many new parameters into the nucleosynthesis problem enabling one, for example, to fit the observed light element abundances without constraining the number of neutrino species

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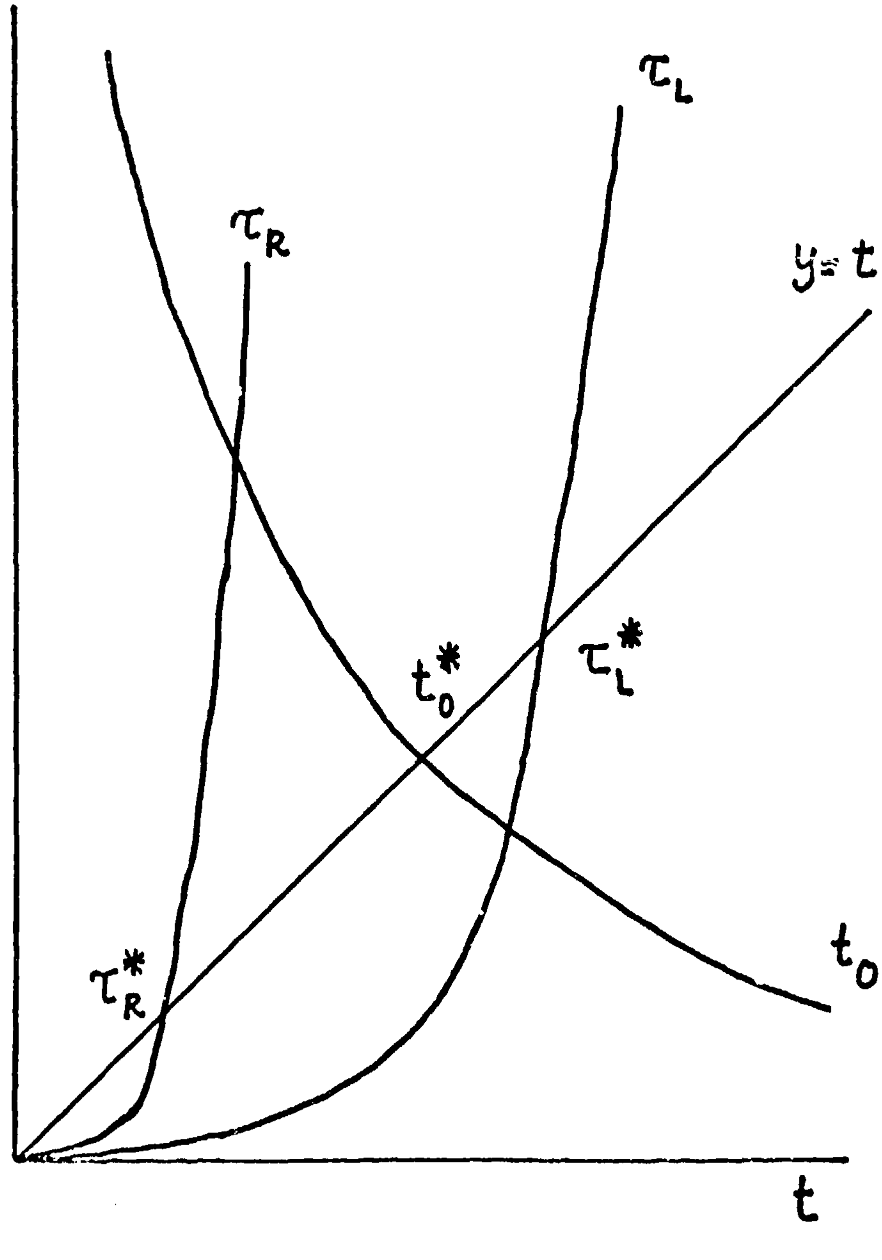

Fig. 1: Development of the characteristic times associated with neutrinos with the evolution of the universe.